On-line fatigue alleviation for wind turbines by a robust control approach

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ABSTRACT

This paper proposes a sliding-mode based robust control technique aimed at fatigue alleviation of a Wind Energy Conversion System (WECS). The control architecture incorporates an on-line fatigue estimator, which can be used as a virtual sensor of the fatigue damage in the feedback control loop. This virtual sensor allows to evaluate and predict the potential fatigue damage by on-line processing signals such as the tower top acceleration (typical experimental acquisition) or the tower base bending moment (typical numerical measure). The output of the fatigue virtual sensor is fed to a robust controller aimed at reducing loads of the wind turbine components, and consequently fatigue stresses, by properly modulating the pitch angle. The proposed control solution has been validated on the National Renewable Energy Laboratory (NREL) 5-MW three-blade wind turbine model.

1. Introduction

Fatigue damage is the weakening or breakdown of a material subject to repeated series of stresses. Fatigue occurs in a microscopic scale, manifesting itself as deterioration or damage in components or structures. A detailed history of fatigue can be found in [1]. Fatigue is generally estimated using the rainflow counting (RFC) method [2], which is used in combination with the Palmegreen-Miner rule of linear damage accumulation [2]. The RFC method is still the basis for theoretical damage estimation but has an algorithmic nonlinear structure requiring a significant history window, and for this reason is used mainly as a post-processing tool.

An important design goal for large Wind Turbines (WTs), beyond to achieve robust control performance under different operating conditions [3,4], is to reduce fatigue and extreme loads on support structure and blades by control. In this context, most of the current control methods are based on the minimisation of certain norms of the stress on different components of the wind turbine, which are expected to reduce fatigue, but are not a reliable characterisation of the damage. In [5] a tower dampening control strategy is proposed based on an individual blade pitch control architecture that employs an estimate of the tower fore-aft velocity based on measurements from blade load sensors. Additionally, accelerometers and load sensors have been used to control the pitch of each blade independently to achieve reductions in loading, and in [8] the control method adjusts the pitch angles using the measure of the local inflow angle and relative velocity on each of the blades. Indeed, all the above mentioned control strategies result in fatigue load reductions, but the damage can be only estimated offline.

The use of wind speed measurements based on laser imaging detection and ranging (LIDAR) to reduce turbine fatigue loads is an area that is being actively researched. Considering LIDAR assisted control, disturbance feedforward controllers for load reduction have been proposed in [9,10] to assist collective pitch control and in [11–13] to improve individual pitch controllers. Also with the above controller the damage is estimated a posteriori. In [14–16], LIDAR sensor is utilized to measure in advance the coming wind in order to optimize the control operation of model predictive control (MPC) schemes for the time horizon during which the wind is measured. The main drawback of MPC is the large computation time required in each step to solve the optimization problem, and the effectiveness in load reduction of the above approaches is just verified offline by a simple comparison with conventional controllers. In [17] an MPC control strategy for wind farms has been developed with the goal of dispatching power demands between WTs to minimize tower bending and fatigue on the motor shaft. Also in this last approach, the estimated damage fatigue is not used as feedback variable.

In contrast to the previously discussed papers, where the evaluation of the fatigue behaviour of wind turbines is performed off-line (by post processing experimental acquisitions or simulation results), a data-
based MPC strategy that incorporates an on-line fatigue estimation method through the objective function has been proposed in [18], but considering a linearization of the WT model around a specific operating point. An approximated linear time invariant (LTI) model is used in [19,20] to design linear quadratic regulator (LQR) controllers with different load reduction capacity. The switching between the controllers is empirically triggered by different levels of the accumulated damage. To the best of the authors’ knowledge, only the above articles [18–20] use fatigue damage as feedback variable, but the considered control laws have been designed based on a linear approximations of the wind turbine, without taking into account model uncertainties and external disturbances. The added value of the present study consists in considering the uncertain nonlinear model of a WECS, coupling an on-line fatigue estimator with a nonlinear controller based on Sliding-Mode Control (SMC) theory, renowned for its robustness with respect to matched perturbations and easy implementability. In particular, the aim of the present paper is to design an online slide-mode pitch control technique, based on on-line fatigue estimation, reducing structural loads for WECS operating in partial load region. Blade pitch and generator torque are manipulated to achieve the conflicting objective of maximising power production and load reduction [19]. The main features of the considered strategy can be summarised as follows: (i) implementation of an on-line fatigue damage estimation algorithm for control purposes; (ii) proposal of a least squares approximation technique of the aforementioned damage estimation; (iii) design of a robust control strategy for wind turbine fatigue damage reduction using the approximate damage estimation as a virtual fatigue sensor.

The paper is organized as follows. Section 2 describes the proposed on-line method for fatigue damage estimation. Section 3 gives the WECS dynamics and Section 4 presents the proposed robust control strategy for fatigue damage reduction. Simulation results and conclusions are presented in Sections 5 and 6, respectively.

2. On-line fatigue sensing

The most common evaluation method of the fatigue behavior, i.e. of the damage, requires two steps: to choose a counting and identifying method for the alternating cycles of the signal under examination and to identify a damage model together with a fatigue strength curve. As recalled in the introduction, the most recognized and used counting method is RFC [2]. It has a complex sequential structure in order to decompose arbitrary sequences of loads into cycles. To compute a lifetime estimate from a given structural stress input, the RFC method [2] is applied by counting cycles and extrema, followed by the Palmgren-Miner rule [2] application together with the adoption of a material-specific fatigue strength curve (i.e. S-N, Stress amplitude vs. cycles Number curve) to calculate the expected damage [1]. The RFC method is a nonlinear numerical algorithm and not a mathematical function; thus it can only be used as a post-processing algorithm. Hence, it is not possible to use RFC for real-time control, since it requires a time series of stress and not only instantaneous measurements, as it is the case in feedback control loops. In contrast to the RFC method the approach developed in this paper has the advantage that it can be incorporated on-line in the feedback control loop. In the following the standard (Section 2.1) and the proposed (Section 2.2) evaluation of the damage time history is described. In Section 2.3 the on-line parameter estimation for fatigue sensing is given.

2.1. Standard evaluation of damage time history

If a generic signal (e.g. acceleration, force, moment) is considered to evaluate the mechanical system or component fatigue behavior, to estimate its durability performance the fatigue strength S-N curve related to it has to be known and its expression is the following, similar to the Wöhler [2] curve for stress signals:

\[ x_n = \alpha n^\gamma \]  

(1)

where \( x_n \) is the strength amplitude value of the signal related to an applied cycles number \( n \), \( \alpha \) is the intercept of the curve on the amplitudes axis for \( n = 1 \), \( \gamma \) is the curve slope considered constant in the whole cycles range.

Its inverse representation is also valid:

\[ N = \left( \frac{x_n}{\alpha} \right)^{1/\gamma} \]  

(2)

where \( N \) represents the strength cycles number when an amplitude value \( x_n \) of the alternating signal is applied.

After having defined the fatigue strength curve, the fatigue behavior evaluation method needs to count cycles and to apply a damage model (i.e. Palmgren-Miner [2]). Regarding the cycles counting, the counting method considered as standard in this paper, but considered as such by the scientific community and by international standards, is the rainflow counting [2]. RFC identifies the closed hysteretic cycles defined by the signal and, generally, the cycles are collected in bands (bins) to reduce the result dimensions of this evaluation. A load spectrum, that is a three-column matrix, can be obtained in which the number of counted cycles \( n \), the associated mean value \( x_n \) and the amplitude value of the signal \( x_n \) are represented in its generic row. All the counted cycles can also still kept in memory, with relative amplitude and mean value, without to be sampled in bands, obtaining, in this case, a spectrum with as many rows as many cycles were counted, that is assuming for each row \( n = 1 \).

The presence of a mean value would require a further step in order to adopt the previously mentioned damage model. By adopting, for example, the correction of Goodman or Gerber [2] it is possible to trace back to an equivalent amplitude value of the cycle by knowing \( x_{n_{min}} \), that is the ultimate static strength related to the variable. But, the generic signal \( x \) that is going to be analyzed (e.g. acceleration) does not always allow to go back to parameters strictly related to the component strength, for example to the ultimate static strength \( x_{n_{min}} \), for this reason the first simplification hypothesis assumed is that the mean value of the generic cycle will be neglected.

Assuming the above hypothesis the load spectrum can be represented as:

\[ (x_n, n) \]  

(3)

where \( x_n \) an \( n \) are the vectors of amplitude and the number of applied or counted cycles, respectively. By knowing the spectrum of Eq. (3), fatigue damage is evaluable by the Palmgren-Miner rule:

\[ D_p = \sum_{i=1}^{m} \frac{n_i}{\left( n_{min} \right)^{1/\gamma}} \]  

(4)

where \( m \) is the total number of counted cycles, \( D_p \) the cumulated damage [2]. Subscript \( p \) is used to remember that the damage, not being calculated necessarily starting from a stress value, is a potential damage [21,22], very useful for comparative analysis but not to be analyzed as the absolute value of the real damage.

Another definition, useful to better understand the subsequent steps proposed by the method object of the present paper, is that of Damage Equivalent Signal (DES) [23], often used in the field of wind engineering.

Under the hypothesis of constant slope of the fatigue strength curve, by knowing the damage or equivalently the load spectrum, it is possible to define a stationary cyclic condition equivalent to the entire spectrum [24] in terms of damage. Given an arbitrary number of cycles, to which it is possible to assign the value of the total number of cycles \( m \), it is always possible to evaluate the equivalent amplitude value \( x_{n_{eq}} \) of the signal which determines the same damage of the spectrum \( (x_n, n) \) by means of the following equation [2]:
that can be also expressed as follows by adopting the damage definition of Eq. (4):

\[ x_{\text{des}} = \sum_{i=1}^{m} \left( \frac{m_i}{D_p} \right)^y. \]  

(6)

The evaluation of the cumulative damage at a given moment in the life of the mechanical system requires to acquire the whole history of the signal, considered representative of its behavior, meaning as whole the one that goes from the first use of the machine, seamlessly, up to that moment.

This ideal approach is impossible to be followed both for reasons of memory space allocation and considering the computational times necessary to count cycles through the RFC and then to evaluate damage. To have a reference value of damage useful for the development of control strategies towards damage the authors have developed a procedure that follows the standard evaluation previously defined.

This procedure needs a signal measured throughout its temporal extension \( T \). In Fig. 1 the flow chart of the ideal procedure for damage evaluation is shown. The real potential damage is evaluated by applying the RFC and Palmgreen-Miners rule of Eq. (4) in increasing time intervals \([0, t_i] \), with \( t_i \) that increases from 0 to \( T \) as described by the left plot of Fig. 1. This will allow to define a time history of the damage \( x_{\text{des}}(t) \) as the right plot of Fig. 1 shows. Correspondingly, the time history of the damage equivalent signal \( x_{\text{des}}(t) \) can also be obtained. The described algorithm will be the reference procedure used in the following to obtain the so called “real damage”, used to evaluate the control performance. Concerning cycle counting, the formulation of the RFC method relies on the hypothesis that either the signal time history is single or repeats itself several times (random signal hypothesis). In our case each evaluation in the window \([0, t_i] \) has been performed adopting the hypothesis of single time history. Moreover, the proposed reference procedure does not consider cycles mean value.

2.2. Proposed evaluation of damage time history

The authors idea is to monitor the potential damage of a generic machine by evaluating it at any of the operating times without taking up all the memory space required by the ideal methodology, evaluating it by adopting a floating window \( \Delta T \) defined in the time domain, of appropriate characteristics. Let’s imagine now to observe only this floating window and to apply the standard procedure to it. Once the window has been defined, this will be the data buffer that will continuously be filled in for the evaluation of fatigue behavior. In Fig. 2 the flow chart of the procedure proposed by authors is shown.

When the mobile i-th window \( \Delta T_i \) is post processed the load spectrum obtained by the RFC is:

\[ (x_{n, m}, n), \]  

(7)

As concerns cycle counting, in this case, by authors hypothesis, each mobile window signal is considered as random and, by adopting the rule of Cloorman-Seeger or ASTM [25,24], all the cycles are forcibly closed.

If a strength curve such as Eq. (1) is adopted, it is possible to define the i-th potential damage \( d_{pi} \), that will be called instantaneous damage, meaning by instantaneous the one associated to the current mobile window:

\[ d_{pi} = \sum_{k=1}^{m_i} \left( \frac{m_i}{D_p} \right) \]  

(8)

in which subscript \( i \) refers to i-th window and \( k \) to the generic spectrum cycle of Eq. (7), counted in the same window. The term \( m_i \) is the total number of cycles counted in the window.

The cumulated damage at the generic instant, that is at the generic i-th window, will be:

\[ D_p = \sum_{i=1}^{i} d_{pi}, \]  

(9)

Similarly, the DES related to the window is:

![Fig. 1. Flow chart of standard evaluation of damage time history.](image-url)
\[ x_{\text{des}} = \alpha \left( m_i - \sum_{k=1}^{m_i} \left[ \frac{\Delta \tau}{(n_i)} \right]^Y \right) \]

(i.e., considering Eq. (8))

\[ x_{\text{des}} = \alpha \left[ \frac{m_i}{d_{p_i}} \right]^Y. \]  

The value \( x_{\text{des}} \) is strongly influenced by the number of cycles counted in the window, \( m_i \), and therefore window by window, could vary in value, increasing or decreasing, without, however, meaning that the damage has really increased or decreased. For example, if two windows \( i - \theta \) and \((i + 1) - \theta \) generate the same instantaneous damage \( d_p \), but the two windows contain different numbers of cycles \( m_i \) and \( m_{i+1} \), two different values of \( x_{\text{des}} \) occur for the same damage. To overcome this result and have a value of \( x_{\text{des}} \) comparable among the various windows and, therefore, independent of the number of cycles, the value of the normalized DES has been defined \( x_{\text{des}} \), that is evaluated in the hypothesis of a number of cycles constant for all the windows. In the case of number of cycles constant and equal to 1, Eq. (11) becomes:

\[ x_{\text{des}} = \alpha \cdot d_{p_i}^{-Y}. \]  

2.3. On-line parameter estimation for fatigue sensing

As stated in the introduction, the aim of this paper is to control the power of a wind turbine, while reducing the incurred fatigue. However, inclusion of the fatigue reduction in the control system is not a simple task, because of the non-linearities introduced by the damage calculation.

Therefore the fatigue calculation will be done by a fatigue estimator in the least squares sense, where the identification scheme has the tower top fore-aft velocity as an input, i.e. \( x_{\text{des}} \) (see Eq. (15)), and the normalized DES given by Eq. (12) \( (x_{\text{des}}(t_k)) \) as output, where \( t_k \) is the time instant. The goal of the estimation scheme is to approximate the value of the normalized DES given by Eq. (12), considering as signal used to evaluate the fatigue behavior the tower base fore-aft bending moment.

The estimation of the parameters in the least squares sense has been carried out using a recursive ARX model [26] for a dynamical system with one output, that is, \( \delta_k(t_k) = x_{\text{des}}(t_k) \), and one input, i.e., \( x_{\text{des}}(t) \). The ARX model was chosen because of its simplicity, the small number of parameters to be estimated, and the fact that it is consistent with the proposed control scheme formalism. Therefore the following ARX model has been used:

\[ A(q^{-1})\delta_k(t_k) = B(q^{-1})x_{\text{des}}(t_k) - \eta_k \]  

where \( q^{-1} \) is the backward shift operator, \( B(q^{-1}) = b_0 q^0 \) gives the desired parameters \( b_0 \) to be online estimated and \( A(q^{-1}) = 1 + a_0 q^{-1} \) with \( a_0 = -1 \), accounts for the embedded integrator in the \( x_{\text{des}} \) operator, \( \eta_k = 0 \).

3. WECS dynamics

For the nonlinear reduced model of the turbine used by the model-based controller, the disturbance is reduced to the rotor effective wind speed and only three degrees of freedom are considered [16]. The first tower fore-aft bending mode, the rotational motion and the collective pitch actuator are based on [27,16]:

\[ J_\omega \dot{\omega}(t) = T_i(t) - T_e(t) N_g \]  

(14)

\[ m_1 x_1(t) + c_1 x_1(t) + k_1 x_1(t) = F_0 \]  

(15)

\[ \ddot{\beta}(t) + 2\zeta_0 \omega_0 \dot{\beta}(t) + \omega_0^2 \beta(t) - u(t) = 0 \]  

(16)

The first equation models the drivetrain dynamics, where \( \omega(t) \) is the rotor speed, \( T_i(t) \) is the aerodynamic torque and \( T_e(t) \) the electrical generator torque, \( x_1(t) \) is the tower top fore-aft displacement, \( \beta(t) \) is the effective collective blade pitch angle. Moreover, \( N_g \) is the gear box ratio, and \( J \) is the sum of the moments of inertia about the rotation axis of the rotor hub \( J_{hi} \), blades \( J_B \) and the electric generator \( J_G \) [16], i.e \( J = J_{hi} + 3J_B + J_G N_g^2 \). The second equation describes the tower fore-aft dynamics. \( F_0 \) is the aerodynamic thrust, and \( m_r, c_r \) and \( k_r \) are the tower equivalent modal mass, structural damping and bending stiffness, respectively [16]. Finally, the third equation is a second-order model of the blade pitch actuator, where \( u(t) \) is the collective blade pitch control input, \( \sigma \) is the undesamped natural frequency and \( \xi \) is the damping factor.

The aerodynamic power that the wind turbine extracts from the wind is expressed by the following equation [28]:

\[ P_{\text{wind}}(t) = \frac{1}{2} \rho \pi R^2 \cdot C_{p,\text{hub}} \cdot \left( \frac{\omega(t)}{\omega_0} \right)^2 \left( 1 - \left( \frac{\omega(t)}{\omega_0} \right)^{-3} \right) \]  

where \( \rho \) is the air density, \( R \) is the radius of the turbine, \( C_{p,\text{hub}} \) is the hub aerodynamic power coefficient, \( \omega_0 \) is the rated rotor speed and \( \omega(t) \) is the rotor speed. The power extracted increases quadratically with the wind speed up to the rated value, and then it remains constant.
\[ P_c = \frac{1}{2} \rho \pi r^2 C_p(\lambda, \beta) V_{ref}^3 \]  

(17)

where \( \rho \) is the air density, \( r \) is the wind turbine rotor radius, \( V_{ref} = V_0 - \Delta V \) is the relative wind speed and \( C_p(\lambda, \beta) \) is the power coefficient, depending on both the blade pitch angle \( \beta \) and the tip speed ratio \( \lambda \), defined as \( \lambda = \frac{r \omega}{V_0} \) \[29\]. Therefore, the torque that the WT extracts from the wind is:

\[ T_a = \frac{\rho \pi r^2 C_p(\lambda, \beta) V_{ref}^3}{2 \lambda}. \]  

(18)

The power coefficient \( C_p(\lambda, \beta) \) is a nonlinear function characterizing the efficiency of the energy transfer from wind energy to mechanical energy \[30\], and depends on \( \lambda \) and \( \beta \).

The aerodynamic thrust acting on the rotor with radius \( r \) has the form

\[ F_t = \frac{1}{2} \rho \pi r^2 C_t(\lambda, \beta) V_{ref}^2, \]  

(19)

where \( C_t \) is the effective thrust coefficient. The \( C_t \) and \( C_p \) coefficient can be described by the expression \((i = t, p)\):

\[ C_i = C_{i0} \left( \frac{1}{\lambda + 0.08 \beta} - \frac{0.035}{(\beta^2 + 1)} \right) + k_{i1} \beta + k_{i2} \exp \left( \frac{1}{\lambda + 0.08 \beta} - \frac{0.035}{(\beta^2 + 1)} \right) \]  

(20)

obtained as nonlinear function fit to two-dimensional lookup tables which can be obtained from the NREL 5-MW wind turbine generated using the NREL code WT perf \[31\]. With reference to the thrust coefficient, the previous expression can be approximated considering a linear polynomial fit of the form

\[ C_t = p_0 + p_1 \lambda + p_2 \beta \]  

(21)

with \( p_0 = 0.6226, p_1 = 0.022551, p_2 = -0.072420 \) in the range \( \beta \in [0, 10] \) deg and \( \lambda \in [7, 9] \).

Mechanical parameters are often uncertain. To show the robustness of the proposed approach, some of them will be assumed affected by unknown but bounded uncertainties:

**Assumption 3.1.** Parameters \( c_T, m_T \) and \( k_T \) appearing in (14) and (15) are uncertain, with bounded uncertainty:

\[ c_T = c_T^0 + \Delta c_T; \quad m_T = m_T^0 + \Delta m_T; \quad k_T = k_T^0 + \Delta k_T \]

\[ |\Delta c_T| \leq \rho_c; \quad |\Delta m_T| \leq \rho_m; \quad |\Delta k_T| \leq \rho_k \]

\( \rho_c, \rho_m, \rho_k \) being bounded known positive constants.

4. Fatigue alleviation

The proposed control architecture, addressing wind turbines operating in the region of partial load, is constituted by the baseline generator-torque controller implemented as described in \[32\] and a novel collective pitch controller aiming at reducing the fatigue damage. As noted in \[33\] more than half of the annual energy capture of the modern wind turbine is said to occur within the region of partial load.

In this region the aim of the control is to optimize power capture, and in most of maximum power point tracking (MPPT) control methods the baseline generator-torque controller is considered when the blade pitch angle is held at a constant optimum value \( (\beta_{opt}) \) that yields the maximum aerodynamic lift. In this control method, the generator torque is proportional \( (K_{mp}) \) to the square of the generator speed to maintain a constant \( (opt) \) tip-speed ratio \( (\lambda_{opt}) \). For the considered 5-MW NREL wind turbine \[32\], the peak power coefficient \( (C_{p_{max}}) \) of 0.482 occurs at a tip-speed ratio \( (\lambda_{opt}) \) of 7.55 and a rotor-collective blade-pitch angle \( (\beta_{opt}) \) of 0.0 deg. With the 97:1 gearbox ratio, the optimal constant of proportionality \( (K_{mp}) \) is 0.0255764 Nm/rpm\(^2\) in the region of partial load control law. The generator torque is saturated \((T_{ref, sat})\) to a maximum of 47402.91 Nm, and a torque rate limit \((T_{ref, limit})\) of 15,000 Nm/s is imposed.

In this paper to achieve the conflicting objective of maximising power production and load reduction \[19\], a second control loop has been added to the above MPPT control method. In particular a pitch controller is designed as in the following to reduce the tower base fore-aft bending moment. To be noted that the blade pitch control signal \( (\beta) \) is limited between the optimal pitch angle \( (\beta_{opt} = 0 \ deg.) \) and 10 deg in order to avoid large excursion of the generator torque leading to power production reduction.

To design the pitch controller, consider the identified model structure of Section 2.3 which has the form

\[ \delta_i(t) - \delta_i^0(t) = b_i(t) \bar{x}_i(t) \]  

(22)

Dividing by the sampling time \( T_i \), one gets

\[ \frac{\delta_i(t) - \delta_i^0(t)}{T_i} = \frac{b_i(t)}{T_i} \bar{x}_i(t) \]  

(23)

which can be considered as an approximation of

\[ e(t) = \delta_i(t) - \delta_i^0(t) \]  

(25)

where \( \delta_i^0(t) \) is a constant reference variable, and the corresponding sliding surface:

\[ s(t) = e(t) + \mu e(t) = 0 \]  

(26)

with \( \mu > 0 \). As well known, the establishment of a sliding motion on the surface \( s(t) = 0 \) ensures the asymptotical vanishing of the error variable with dynamics depending on the choice of \( \mu \) \[34\]. According to the classic formulation \[34\], the attainment of such sliding motion in finite time is achieved by imposing the condition \( s(t) \bar{s}(t) \leq -\gamma |s(t)| \), with \( \gamma \) arbitrary positive constant, robustly with respect to bounded matched uncertain term.

In order to impose the above condition, the quantity \( s(t) \bar{s}(t) \) needs to be computed for the identified system \( (24) \):

\[ s(t) \bar{s}(t) = s(t)(\bar{s}_i(t) + \bar{\mu}(t) \bar{x}_i(t)) \]

\[ = s(t) \left( \bar{b}(t) \left[ \frac{\bar{\mu}}{m_T} + \frac{\bar{b}(t)}{b(t)} \bar{x}_i(t) \right] - \frac{k_T}{m_T} \bar{x}_i(t) \right) \]

\[ + \frac{n_T^2 V_{ref}^2}{2 m_T} \left( c_T(\lambda, \beta) \right) \]  

(27)

Using the approximation (21) for the thrust coefficient, one gets:

\[ s(t) \bar{s}(t) = s(t) \left( \bar{b}(t) \left[ \frac{\bar{\mu}}{m_T} + \frac{\bar{b}(t)}{b(t)} \bar{x}_i(t) \right] - \frac{k_T}{m_T} \bar{x}_i(t) \right) \]

\[ + \frac{n_T^2 V_{ref}^2}{2 m_T} \left( p_0 + p_1 \lambda + p_2 \beta \right) \]  

(28)

i.e.:

\[ s(t) \bar{s}(t) = s(t) \left( \bar{p}_0 \bar{b}(t) \frac{n_T^2 V_{ref}^2}{2 m_T} \left( p_0 + p_1 \lambda + p_2 \beta \right) + \bar{\beta}(t) \bar{\beta}(t) \bar{s}(t) \bar{s}(t) \right) \]  

(29)

where the last term contains uncertain terms according to Assumption 3.1 and reads.
\[
\delta(t, x_i(t), \dot{x}_i(t)) = \left(\frac{\mu - \frac{C_T}{m_T} + \frac{b_i(t)}{b(t)} k_T}{p_2 n_T r V_{\text{ref}}^2} x_i(t) - \frac{k_T}{m_T} x_i(t)\right)
\]

In order to find a bound for the uncertain term (30), define:

\[
b_M = \max_{t \geq t_0} \left\{ \frac{b(t)}{b(t)} \right\}
\]

and

\[
\delta(t, x_i(t), \dot{x}_i(t)) = 2(m_T + \rho_m)\left(\frac{\mu - \frac{C_T}{m_T} + \frac{b_i(t)}{b(t)} k_T}{p_2 n_T r V_{\text{ref}}^2}\right) x_i(t) + \frac{\mu_x + \rho_x}{m_T} x_i(t)
\]

Therefore the uncertain term (30) can be bounded according to:

\[
|\delta(t, x_i(t), \dot{x}_i(t))| \leq \delta(t, x_i(t), \dot{x}_i(t)) \forall t \geq t_0
\]

According to the classic formulation, the attainment of a sliding motion upon the sliding surface \(s(t) = 0\), and hence asymptotic vanishing of the tracking error, is guaranteed by the following control law:

\[
\beta(t) = \beta_0(t) + \beta_0(t)
\]

with:

\[
\beta_0(t) = -\frac{b_0 + p_1}{p_2} x_i(t)
\]

(35)

\[
\beta_0(t) = \left[\delta(t, x_i(t), \dot{x}_i(t)) + \frac{\beta_0}{p_2 n_T r V_{\text{ref}}^2} \right] \cdot \text{sign}(s(t))
\]

(36)

with \(\beta_0 > 0\). The above control law guarantees that the sliding mode existence condition is verified, i.e.:

\[
s(t) \leq \frac{\beta_0}{2(m_T + \rho_m)} \cdot \left| s(t) \right|
\]

(37)

this ensuring that the tracking error (25) asymptotically vanish (i.e. the variable \(\delta_i(t)\) asymptotically tracks the constant reference variable \(\delta_i^{\text{ref}}\) with dynamics determined by the constant \(\frac{\beta_0}{2(m_T + \rho_m)}\).

5. Simulation results

The proposed fatigue reduction sliding mode based controller, coupled with the baseline generator-torque control system as described in Section 4, has been tested by intensive simulations. The block diagram of the overall controller structure, hereafter called FSC, is shown in Fig. 3. The FSC has been compared with the baseline generator-torque control system (BLC) implemented as described in [32] for wind turbines operating in the region of partial load. The main parameters of the considered WECS have been derived from [16,32] for the NREL 5-MW wind turbine (see Table 1). In particular the plant model (see Section 3) considers the rotational motion, the first tower fore-aft bending mode and the collective pitch actuator [16,27].

Validation tests have been performed using different FAST wind data shown in Fig. 4. In particular, wind speeds of mean value 8 m/s, 9 m/s and 10 m/s have been considered.

As concerns fatigue parameters, the strength curve of Eq. (1) with parameters \(\alpha = 3.7681 \times 10^9[Nm]\) and \(\gamma = -0.3208\) has been considered. Relating to the proposed method of Fig. 2, a mobile window \(\Delta T = 5\) s has been considered. The controller has been designed considering a reference normalized DES \(\delta_i^{\text{ref}} = 2 \times 10^3 [Nm]\) (see Eq. (25)).

Table 1

<table>
<thead>
<tr>
<th>Main WECS parameters.</th>
</tr>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Rotor radius</td>
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<tr>
<td>Hub inertia on low-speed shaft</td>
</tr>
<tr>
<td>Blade inertia on low-speed shaft</td>
</tr>
<tr>
<td>Generator inertia on high-speed shaft</td>
</tr>
<tr>
<td>Gear box ratio</td>
</tr>
<tr>
<td>Tower equivalent modal mass</td>
</tr>
<tr>
<td>Structural damping</td>
</tr>
<tr>
<td>Bending stiffness</td>
</tr>
<tr>
<td>Undamped natural frequency of the blade pitch actuator</td>
</tr>
<tr>
<td>Damping factor of the blade pitch actuator</td>
</tr>
<tr>
<td>Air density</td>
</tr>
<tr>
<td>Peak power coefficient</td>
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<tr>
<td>Optimal tip-speed ratio</td>
</tr>
<tr>
<td>Optimal blade pitch angle</td>
</tr>
<tr>
<td>Optimal constant of proportionality</td>
</tr>
<tr>
<td>Rated generator torque</td>
</tr>
<tr>
<td>Torque rate limit</td>
</tr>
</tbody>
</table>

(interpolated using a zeroth order hold applied to obtain a continuous signal), proving the good performance obtained by the on-line parameter estimation procedure described in Section 2.3. Fig. 6 illustrates the evolution of the controlled fatigue index, i.e. the normalized DES of Eq. (12), obtained with the proposed control architecture (FSC) compared with the BLC controller. The tower base fore-aft bending moment and the output power are shown in Figs. 7 and 8 respectively, for both the baseline generator-torque strategy (BLC) and the proposed fatigue reduction control strategy (FSC). The pitch angle is reported in Fig. 9 for the FSC case compared with the BLC controller.

Fig. 6 shows that the proposed FSC strategy effectively achieves a reduction of the normalized DES with respect to the baseline generator-torque strategy (BLC). Indeed, the proposed control strategy (FSC)
alleviates the mechanical fatigue by reducing the tower base fore-aft bending moment (see Fig. 7). It can be verified that the output power moderately decreases with the proposed control strategy (FSC), as shown in Fig. 8.

To further investigate on the effectiveness of the proposed strategy (FSC), standard equivalent damage loads described in Section 2.1 have been calculated and compared with the baseline generator-torque strategy (BLC). Fig. 10 shows that a damage reduction is achieved by the proposed control method if compared against the baseline generator-torque strategy. The reason for this additional check is that two important assumptions have been introduced in order to incorporate the fatigue estimator as a virtual sensor into the sliding mode architecture. Namely, the effectiveness has been assumed of the use of a floating windows to monitor potential damage (see Section 2.2), and...
the goodness of the fatigue approximation by least squares (see Section 2.3) has been hypothesised. The validity of these assumptions is also confirmed in Fig. 11 where the cumulative damage time history has been compared for the standard (Section 2.1) and the proposed approach (Section 2.2), considering the designed control strategy (FSC).

6. Conclusions

In this paper a fatigue damage estimation technique has been detailed, and used in an on-line fatigue estimation for control. In particular a fatigue damage reduction sliding mode strategy has been
designed. This strategy was implemented on a nonlinear model based on the standard NREL 5-MW wind turbine and it was compared against a baseline generator-torque strategy, achieving damage reduction by reducing the tower base fore-aft bending moment. The results indicate that the proposed control strategy can make a reasonable compromise between power optimization and structural load reduction, guaranteeing extended operational lifetime without much compromise on power maximization objective. Lastly, the approach presented in this paper, allows a control strategy to be implemented in real-time control, effectively incorporating the damage of the components as a virtual sensor into the feedback control formulation.
Fig. 10. Standard equivalent cumulative damage. FSC: blue continuous line; BLC: dashed red line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 11. Cumulative damage time histories coming from WT model controlled by FLC. Proposed evaluation tool - Fig. 2 (blue continuous line); Standard evaluation tool - Fig. 1 (dashed red line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
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References